Theoretical aspects of ERa, the fastest practical suffix tree construction algorithm

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Text indexing problem

Problem statement

Given unstructured input text $T$ consisting of $n$ characters from alphabet $\Sigma$ build an index such that for query pattern $P$ we:

1. determine whether $P$ occurs in $T$ in time $O(|P|)$,
2. find all occurrences of $P$ in $T$ in time $O(|P| + \text{occ})$,
3. find the longest common prefix (LCP) of $P$ and any suffix of $T$ in time $O(\text{LCP}(P, T))$.

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Suffix tree and suffix array (SA) with LCP information are fundamental data structures for indexing unstructured text.
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Huge gap between the theoretical and practical results!

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**Motivation**

Huge gap between the theoretical and practical results!

Analyse the fastest practical algorithm and compare it to the theoretical ones.
Suffix tree

T = ABRAKADABRA$
Parallel External Memory model (PEM)\(^2\)

- Shared-memory model describing well the modern multi-core architecture.
- Two-level memory hierarchy w/ fast&limited private caches on 1\(^{st}\) level and slow&unlimited main memory on the 2\(^{nd}\).
- Features:
  - \(P\) — # of processing elements
  - \(B\) — block size
  - \(M\) — cache (main memory) size

\(^2\)Arge, Goodrich, Nelson, Sitchinava (2008)
Parallel External Memory model (PEM)

- We assume CREW PEM.
- Data are read and stored on disk and not fitting into main memory:
  - Denote first level as the main memory and second level as the disk.
- Performance metrics:
  - parallel time,
  - parallel block transfers bw/ disk and main memory.
SUFFIX TREE CONSTRUCTION LOWER BOUNDS

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\(^3\)EM model

\(^4\)Uncompressed index

\(^5\)PEM model
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- Yet, it’s fast in practice: Constructs and stores the human genome’s suffix tree in 14 minutes on 16-core desktop PC w/ HDD storage!
- Key idea:
  - DNA, music, proteins are almost random
  - Suppose random input text and PEM model of computation:
    - What is expected time complexity?
    - What is expected I/O complexity?

---

$^6$Heinz, Zobel, Williams (2002)

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ERa steps

ERa constructs the suffix tree in two steps:

1. The **vertical partitioning** step determines the suffix subtrees just fitting into $M$.

2. The **horizontal partitioning** step builds the actual suffix subtrees.
Define **S-prefix** as the prefix of the suffix in the text.
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   - Note: We obtain S-prefixes of length 1.

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3. Repeat step two for S-prefixes of length 3, 4... until all $f_\pi$ fit into the memory $M$.

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4. To optimally fill the main memory combine the S-prefixes into groups fitting into the main memory as tight as possible.
   - Use First-Fit Decreasing heuristic for bin packing problem\(^8\).

---

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Vertical partitioning

Analysis:

- For uniform string distributions $O \left( \log |\Sigma| \frac{n}{M} \right)$ scans.
  - Note: The main memory contains $O \left( \frac{n}{M} \right)$ S-prefixes of total size $O(M)$.
- First-Fit Decreasing heuristic requires $O \left( \left( \frac{n}{M} \right)^2 \right)$ time.
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Overall:

- $O \left( n \log |\Sigma| \frac{n}{M} + \left( \frac{n}{M} \right)^2 \right)$ time.
- $O \left( \frac{n}{B} \log |\Sigma| \frac{n}{M} \right)$ I/Os.
Horizontal partitioning

In parallel, for each group of S-prefixes, construct the suffix subtree:

1. Initialize the optimal length of S-prefixes range $= \mathcal{O}(M/n_M) = \mathcal{O}(M^2/n)$.

Note: The name Elastic Range.

2. Fill buffers with range characters following S-prefixes in the text.

3. Sort them in lexicographic order and remember their branching information (=LCP) and the original position (=SA).

4. While some of the sorted substrings are not unique, repeat steps two and three.

Note: Unique strings' buffers are used for non-unique ones in the next iteration, range is increasing, the frequency dropping.

5. Construct the suffix subtree using depth-first traversal and reading SA and LCP.
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Horizontal partitioning

In parallel, for each group of S-prefixes, construct the suffix subtree:

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   \[ \text{range} = O \left( \frac{M}{\sqrt{M}} \right) = O \left( \frac{M^2}{n} \right) \]
   
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5. Construct the suffix subtree using depth-first traversal and reading SA and LCP.
Horizontal partitioning

Analysis:
- Assuming random string distribution, the number of uniquely represented strings in $O(M)$ space is $|\Sigma|^{range}$
  - The number of step two and three iterations is bounded by $O(\log|\Sigma| M)$.
- Each iteration, filling the buffers is done in $O(M)$ time and $O(M/B)$ I/Os.
- Sorting can be done in $O(M)$ time using in-memory radix string sort and requires no I/O.
- Traversing the suffix subtree requires linear $O(M)$ time and $O(M/B)$ I/Os.
Horizontal partitioning

Analysis:

- Assuming random string distribution, the number of uniquely represented strings in $O(M)$ space is $|\Sigma|^{\text{range}}$.
  - The number of step two and three iterations is bounded by $O(\log |\Sigma| M)$.
- Each iteration, filling the buffers is done in $O(M)$ time and $O(M/B)$ I/Os.
- Sorting can be done in $O(M)$ time using in-memory radix string sort and requires no I/O.
- Traversing the suffix subtree requires linear $O(M)$ time and $O(M/B)$ I/Os.

Overall for constructing $O \left( \frac{n}{M} \right)$ suffix subtrees:

- $O \left( \frac{n}{p} \log |\Sigma| M \right)$ parallel time.
- $O \left( \frac{n}{pB} \log |\Sigma| M \right)$ parallel block transfers.
ERa despite being practically the fastest algorithm is not theoretically tight even for random input strings with uniform substring distribution.

Open problem: Is it possible to design a theoretically tight yet practically competitive algorithm for suffix tree construction?

Future work: Analyse ERa bottlenecks in practice and see if they match the critical terms in time and I/O complexities presented here.
Conclusion

- ERa despite being practically the fastest algorithm is **not theoretically tight** even for random input strings with uniform substring distribution.
- Open problem: Is it possible to design a theoretically tight yet practically competitive algorithm for suffix tree construction?
**Conclusion**

- ERa despite being practically the fastest algorithm is **not theoretically tight** even for random input strings with uniform substring distribution.

- Open problem: Is it possible to design a theoretically tight yet practically competitive algorithm for suffix tree construction?

- Future work: Analyse ERa bottlenecks in practice and see if they match the critical terms in time and I/O complexities presented here.
Thank you.

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